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SOME POINTS IN THE THEORY OF INFERENCE.

IT is the consciousness of the most difficult and intricate processes of thought that is the clearest, and, in proportion as the process becomes easy and simple in itself, so does it become to reflective scrutiny obscure and difficult of recognition. As we might expect, therefore, it is the nature of elaborate and difficult reasoning, such as that made use of in scientific discovery, that has been most fully investigated and is best understood. Such reasoning is the type that we have at hand on which to base our conception of reasoning in general. The investigation of it is in reality the most elementary part of the science of Logic. The most advanced and difficult part is the investigation of the nature of that sort of reasoning which is so simple and rapid that it tends to become organic.

It is thus only of late years that the process by which distance is inferred has come to be recognized as reasoning at all. While Mill recognizes that this is inference, the fact is for him an isolated one, and is not seen to have any ulterior bearings. The general conception which runs through his Logic is that reasoning is one thing and observation another, and that there is no difficulty in drawing a hard and fast line between the two. The conception, however, does not fail to land him in difficulties. He is driven by it to the conclusion that Kepler's discovery of the elliptical path of the planets was not a case of reasoning at all, but merely a case of observation. He compares it, rightly enough, I think, to the process by which a navigator on sailing round a piece of land might conclude that it was an island. As the navigator takes in his facts piecemeal, and then brings them together under the general description 'island,' so Kepler took in the facts of the successive positions of Mars in the same way, and brought them together under the general description of 'ellipse.' It did not occur to Mill, however, that the true conclusion was, not that Kepler's process was not reasoning, but that the navigator's was.

Two cases have to be distinguished, namely, that of the navigator sailing round the land before he had the conception 'island' in his head, and that of his sailing round it afterwards. We may conceive of a navigator born yesterday cruising the high seas and finding here land that he could not anyhow get round, and here again land that he could. As the respective differences and resemblances dawned upon him, we might conceive him naming the one 'continent' and the other 'island.' Thus the conception would be formed and the name attached. We may then suppose him coasting along a piece of land, uncertain whether it is a continent or an island. In the end he succeeds in getting round it, and then we say he *establishes the fact* that it is an island. We thus arrive at the conclusion, — one of considerable interest, I think, — that the process by which in the first instance we constitute a conception is the very same process by which, after the conception is constituted, we prove a fact. If this view is correct, it has an important bearing on questions that lie within the sphere of metaphysics. We prove that anything is real and not an illusion by handling it, and by appealing to those about us, to satisfy ourselves that they can feel and handle it also. Why does this prove to us that the thing is real? Simply because it is in this very manner, by handling things and finding that others concur with us in regard to their existence, identity, and position, that we have in the first instance built up the complex conception of reality.

This, then, may be taken as a specimen of the direct proof of a fact. The navigator takes note of some landmark, a rock or promontory, perhaps; sails round the land till he comes in view of it again; sees the sea all round it; and then pronounces it to be an island. In the process there are three elements: (1) the antecedent knowledge, 'Sea-surrounded lands are islands'; (2) the newly-learned fact, 'This is a sea-surrounded land'; (3) the conclusion, 'This is an island.' The syllogism, then, it seems, is not an idle and meaningless *petitio principii* after all, but accurately enough gives the form that even such elementary reasoning as this must take. The marvellous old Greek was right; it is his critics that have misunderstood the situation. It is perfectly true, of course, that if we look upon the major and minor as something that we knew certainly and definitely to begin with, then the conclusion brings us no new knowledge. The true conception is, however, that while we are reasoning we are alive and moving about, continually learning new facts by observation or testimony, or obtaining new admissions, it may be, from a controversial opponent. The new facts are the minor premiss; the antecedent knowledge is the major. Both are equally essential to the conclusion.

The process of inference, then, naturally falls into two parts: (1) what takes place before the general knowledge evinced in the major is acquired; (2) what takes place afterwards. The first is the ascent from a particular to a general, and so far corresponds to the current conception of Induction. When it takes place, however, prior to the formation of such a concept as 'island,' which has been long embodied in ordinary speech, we do not call it Induction, but Abstraction or Generalization. It is not hard, I think, to see that the two are the same process. When the concept is a more complex one, when, for example, we think of what took place in Newton's mind before the concept of Gravitation sprang into existence, there the process undoubtedly corresponds to what all logicians call Induction. The question, however, next arises: Are we rigidly to confine the term Induction to what takes place before the formation of the concept, to the process that is truly an ascent from particulars to generals? This, it must be said, however appropriate it seems, would not be in accordance with the ordinary practice of logicians. Mill's Inductive Methods, for example, would not in that case be rightly described as Inductive Methods at all. At the same time, if we unite under a common name what happens before the formation of the concept and what happens afterwards, we confuse two mental processes that are markedly distinct. It is the formation of a conception that first brings what is going on in the mind clearly before consciousness. The argument from particulars without a realized general conception is the process of unconscious sagacity. After the conception is formed, such reasoning as then takes place has a definite goal in view. We wish, for instance, to know whether some substance will poison rats, and administer it to ascertain. We do not know in advance, indeed, what Nature's answer will be, but we know that it will be either 'yes' or 'no.' In Induction proper we do not even know this much. Previous to the mental suggestion that gives us the clue to the explanation of the facts, we are simply bewildered. We cannot interrogate Nature on the subject, because we do not know what question to ask. Suddenly the happy thought, as we say, strikes us, and all is clear. It is one noticeable feature of true Induction that it frequently has a charm and interest of which conscious reasoning is devoid. Many of us can listen all day long with unflagging interest to stories of animal or infantile reasoning. The happy thought, the felicitous expression, the slang that 'catches on,' are among the types of the Induction that precedes conception.

Of what description, then, is the reasoning that Mill makes the subject of his Methods? It is plain that it is not that by which general conceptions are formed. It is, however, that by which they are tested. After the formation of the conception all the subsequent reasoning takes the same form. What its nature is, depends on the answer to the question as to what the major premiss is. If that is an ascertained unshakable certainty, then it can be nothing but simple Deduction. If not, it will be that sort of inverted Deduction which Mill identifies with Induction. Let us suppose again that I want to poison rats, and to this end buy some arsenic. I may not even consciously refer to my major premiss, 'Arsenic poisons rats.' The knowledge is there, however, else I would not buy the arsenic. Suppose, however, on the other hand, that I wish to ascertain whether arsenic does infallibly poison rats. I assume for the moment that it does, and administer it. If the rat dies, my assumption is confirmed and becomes for me an established truth. If the rat does not die, and I am certain that what I have administered is arsenic, my assumption is then upset. It is either upset altogether, or it must be taken back and corrected, to be tried again in an amended form. I must next, perhaps, ascertain whether a larger dose will be effective. In this case we have: (1) the Major, 'Arsenic poisons rats,'—an hypothesis to be tested; (2) the Minor, 'This is arsenic,'—an ascertained fact; (3) the formal conclusion, 'This will poison rats.' This formal conclusion is then compared with fact and seen to be wrong, and the real conclusion is arrived at, that the Major premiss will not hold good, at any rate as stated. The form of the sequence in all reasoning is the same. The *dictum de omni et nullo* is valid universally. In such reasoning as Mill applies his methods to, the syllogism is still used, but its use is retroactive. To make the process either of direct Deduction or of this inverse Deduction comprehensible, it seems then that we must assume that there is some new fact learned at the moment, or at any rate before the final conclusion is arrived at, by observation, by testimony, or by admission. In direct Deduction, the new fact is the minor premiss. In inverse Deduction it is the validity or otherwise of the formal conclusion, and this is, in the typical cases, learned by comparison with living fact. Jevons, I think, is right in the opinion that it is when Mill reaches Deduction that his view of reasoning most closely approaches accuracy. In Deduction he finds the three elements: Induction, Ratiocination, and Verification. Where he misses the mark is in not perceiving that the same elements are to be found wherever reasoning exists.

It will be said that, if we identify Induction with Abstraction, we identify two things which have an essential element of difference. An Induction is an assertion, while a concept is not. Perhaps it will be found, however, that this opinion rests on a misconception of the nature of the concept. If we agreed with Professor Huxley that the "houses seen in dreams" were types of the concept 'house,' then indeed it would be impossible to see how a concept could comprise an assertion. That notion of the concept, however, was one which Berkeley thought he had exterminated from philosophy two hundred years ago. One might as well say that those of Turner's pictures which are successful in conveying the effect of a hazy atmosphere are generic, as that a man seen in dreams "like a figure by twilight" is "generic."[1] If we glance at what is called an identical proposition, it will be possible to see that every concept must comprise an assertion. Contrast these two propositions: 'This white thing is not black'; 'White is not black.' The first may not even be true. We might see that the thing in question was black, if we could see it in another light, or if our nerves were in their normal condition. We know that the second must be true, because we at once perceive that we cannot deny it without breaking the law of contradiction, that is, the law that prohibits self-contradiction. If there is a danger of self-contradiction, however, there must have been at the start some assertion for us to contradict. And so there plainly was. 'White is not black,' is not equivalent to 'This white thing is not a black thing,' with which Mill confuses it, and so proceeds to treat it as an ordinary truth of experience. It is equivalent to 'If this thing is white, it is not black,' a conditional proposition which he rightly describes as "a proposition concerning a proposition, the subject of the assertion being itself an assertion." The assertion that we would contradict if we allowed that white was black, is simply the assertion that the name 'white' is applicable to the color 'white' and to nothing else. It is the convention that underlies intelligible speech. It is a truth that we learn, and cannot help learning, whenever we learn the meaning of the words 'white' and 'black.' In regard to simple conceptions such as those of color, it may not seem a truth of much significance, but when we come to conceptions which are a shade more complex, such as those of straightness or equality, we begin to understand its weight.

In holding fast to the Baconian doctrine that we know nothing of the outward world except what we have learned by observation, Mill held fast, I conceive, to an important truth, but he missed in some respects the right method of reconciling this truth with the other facts of the situation. It may be necessary, however, before going further, to meet the inquiry as to whether this is a truth at all. Is it not more in accordance with what recent science has taught us, to hold that we come into the world at birth with a large amount of knowledge of outward things ready made? The pointer puppy points pheasants the first time he is taken out. The monkey has been known to shriek and go into convulsions at the sight of a snake the first time it has seen one. Undoubtedly we do come into the world with innumerable inherited anticipations about the realities that we are afterwards to encounter. The essential characteristic of knowledge, however, is the element of duality, of correspondence between anticipation and fact, and it is of this element that such instances of inherited capacities or sentiments are devoid. They may be true or they may be misleading. We may have an inherited dread of the harmless necessary cat as well as of the serpent. We do not call such anticipations knowledge, at any rate until they have been compared with fact and found valid. We can have no knowledge of matters of fact without the comparison of conceptions with actuality, because knowledge simply means truth ascertained to be truth by reference to fact. Kant's notion that there is in the mind a function for the creation of synthetic propositions *a priori*, is, when translated into plain English, an impossible one. Such propositions are said to add to our knowledge of the world of fact, as contrasted with analytic propositions, which do not. The function would then be one for making up truths in regard to matters of fact, irrespective of all verification by comparison with reality. Such a function as this, we may be sure, no mind either human or divine possesses.

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[1] *Hume*, English Men of Letters Series, p. 98.

Taking it then as established that all truths in regard to matters of fact must rest, in the end, on verification by comparison with fact, either mediately or immediately, we are face to face with the question that is still the crux of metaphysics — the question how it is that we have, in the axioms of mathematics and all that can be deduced from them, a body of truths that present a marked contrast to the ordinary truths of experience, in that we not only know them to be true, but know also that it is impossible for them to be otherwise than true. Mill's answer is, of course, that this fancied knowledge is an illusion, that as our fathers thought the antipodes inconceivable and yet there are antipodes, so, though we think round squares inconceivable, yet there may be parts of the universe where the squares are round and where two and two make five. This, however, is a mere collection of words without a meaning, and can satisfy no one. All that there is of plausibility in it rests on the double meaning of the word inconceivable. No one could ever have had any difficulty in representing to himself antipodes, whatever difficulty he might have had in believing in them. We cannot, however, even represent to ourselves a round square, because we find that precisely in proportion as it becomes round it ceases to be square. In regard to the axiom, 'Two straight lines cannot enclose a space,' Mill says, accurately enough, that the upholders of this proposition maintain it on the ground that we can see its truth by merely thinking of the lines. The answer to this, he thinks, is to be found in the capacity of geometrical forms for being painted in the imagination with a distinctness equal to reality. "Thus," he says, "although we cannot follow two diverging lines by the eye to infinity, yet we know that if they begin to converge, it must be at a finite distance; thither we can follow them in our imagination, and satisfy ourselves that if they approach they will not be straight, but curved." That is an accurate description of the process by which we satisfy ourselves that two straight lines cannot enclose a space; but we cannot help asking: Is it a description of the process by which truths of experience are learned? It is one of Mill's most characteristic doctrines, and one on which he repeatedly and emphatically insists, that "whenever we form a new judgment — judge a truth new to us — the judgment is not a recognition of a relation between concepts, but of a succession, a coexistence, or a similitude between facts."[1] Whether we admit that this applies to all new truths or not, we may certainly admit that it applies to all new truths of experience. It is quite plain that a new truth of experience cannot be learned by the process of comparing one of our concepts with another. Yet it is precisely by doing this, — by comparing our concept of straight lines with our concept of lines that enclose a space, and finding them incongruous, — that Mill describes us as arriving at the truth that they never enclose a space.

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[1] *Examination of Sir William Hamilton's Philosophy*, 4th ed. p. 426.

Are we to take it, then, that so far Hume was right, that the truths of mathematics are truths of idea and of idea only? It seems that they rest on the comparison of concepts with one another, and not with outward fact. That, at all events, is an absolute truth with which any theory on the subject will have to reconcile itself as best it can. Mill's pervading error in dealing with the question of demonstration and necessary truth, is that he treats the lines and circles of geometry as if they were specimens that we had picked up in our rambles, while they really are lines and circles which we suppose ourselves to have just drawn or just constructed. It is to such lines and circles only that Euclid's method of proof is applicable. We take the straight line and make it rotate on a point. We can then deduce all the properties of the circle. The radii are equal, because they are, by hypothesis, the same straight line in different positions. The true postulate of geometry is that the straight line can be placed in any position, and yet be conceived as remaining in all other respects the same. In the fourth proposition, Euclid very plainly postulates that the figures with which he deals should be capable of being lifted and moved about, and placed on top of one another ; in other words, that they be capable of being regarded as identical in spite of difference of position. If this postulate were specifically set forth instead of being tacitly taken for granted, it would be plain that the construction of the fifth proposition and the proof resting on it are mere surplusage. We have in any case to postulate the possibility of taking up the large triangles formed by the extension of the sides, reversing one of them, and placing it on top of the other. Why not at once take up the isosceles triangle, reverse it, and place it on top of its former self? We should thus have two triangles to compare, possessing all the characteristics of those in the fourth proposition. If indeed an attempt were made anywhere among the definitions to define what is meant by the size of an angle, it would be found that no comprehensible definition could be made without rendering this and some half dozen other propositions of the first book self-evident. The equality of two angles, and the equality of two lines joining points respectively equidistant from their vertices, are two expressions for the same fact.

Every geometrical proof rests ultimately on the law of contradiction, on the fact that if we deny its validity we will be found to have denied the assumption with which we started. Are we driven to the conclusion, then, that in dealing with geometrical questions we have to do with something that emanates from ourselves? There seems to be no escape from it. The problem is, how to reconcile such a conclusion with the other indubitable certainty, that the truths of geometry are at the same time truths of the world of fact outside us. The suggestion has been made that they are not, but here I think there is a misconception. The world of fact may not indeed present us with such a thing as an ideally straight line, but it is an important truth for all that, that it presents us with lines that approximate indefinitely to the ideal of straightness, and vary about it as their point of equilibrium. This it is that enables us to make a practical application of the problems of geometry in the world of fact.

The problem, then, of reconciling the proposition that the truths of geometry are truths of idea with the other proposition that they are truths of the outward world, remains to be solved. We find its solution, I think, in this, that the same hypotheses in regard to our own activity which are the subject of Euclid's demonstrations, enter very largely into the constitution of what we call the outward world. If we consider what we really mean by the distance of anything from us, we will find that it is nothing else but the length of the straight line that we would construct if we moved towards it in a uniform direction. If we consider what we mean by its height, we will find that it is nothing else but the length of the line that we would construct if we ascended perpendicularly from its base to its summit. This truth, or something like it, seems to have struck Kant. He observes that "the formative synthesis by which we construct a triangle in imagination is the very same as that which we employ in the apprehension of a phenomenon." The facts of geometry are not singular in that they rest on an hypothesis, on an 'if' and a 'would be.' All facts, indeed, are in the same position. We reckon it the very type of a fact that gold is yellow. As Lotze remarks, however, gold is only what *would be* yellow *if* we saw it in the light.

It is worthy of notice that Hume's definition of necessary truths as truths of idea only gave him, at any rate, a basis for their classification which sufficiently accorded with the current use of language, and did not lead him either into meaningless hairsplittings or into inconsistencies. Kant divided them into two classes, Analytic and Synthetic, and no two philosophers since Kant's time seem to have been able to agree as to the class in which any particular axiom should be included. Mill calls them all Truths of Experience. Bain, however, does not follow him here. He thinks that 'Things which are equal to the same thing are equal to one another' is a truth of experience, but that the proposition, 'Two straight lines cannot enclose a space,' should fall into the identical or implicated class. Mansel, who follows Kant and uses Kantian phraseology, also puts some of the axioms in one class and some in another; but the strange thing is that the very axioms which Bain takes out of the identical class Mansel retains within it, while he excludes others which Bain includes. Huxley would call necessary the certain reminiscence, 'I was in pain yesterday,' and Spencer would at any rate so denominate the statement 'I am in pain now.' Nothing, however, is more certain than that our evidence for the truth of all the axioms of Mathematics, whatever it is, is the same in kind; and, further, that it is of a nature readily distinguishable from our evidence for the truth of matters of fact. Bain, in support of his view that the proposition, 'Things which are equal to the same thing are equal to one another,' is a truth of experience, observes that Equality may properly be defined as immediate coincidence. If it may, then the two terms should be capable of being used convertibly; but it is clear enough that this is not the case. If we predicate equality of two lines, we do not mean that they actually coincide, but that they possess that attribute whereby they would coincide if superimposed. The coincidence of two lines would be a matter of fact to which no necessary proposition could ever apply. Their equality is a matter of conception to which alone such propositions are applicable. The fact of coincidence can be observed by all beings possessed of sight and touch; the notion of equality is probably quite beyond the intellectual grasp of the Bushman or the Dammara. Besides, what can be said about such equality as that between straight lines and crooked ones, equality which is not accompanied with the possibility of coincidence?

If, as I think is clear to demonstration, *a priori* knowledge is a contradiction in terms, then of course there must be a sense in which the truth that things which are equal to the same thing are equal to one another, has its ultimate ground in experience. Where, however, it seems to me that Mill and Bain and the rest of the Empirical school have gone wrong, has been in treating the question as if the experience on which such a truth is grounded were experience obtained after the conception of equality had come into existence, instead of being the experience which had created that conception. If we can conceive of an inductive process by which general laws were arrived at as having taken place antecedently to the formation of such conceptions, then we may be able to understand how it is possible for a whole science to be deduced from them.

Kant, I think, was right in the opinion, — in which, indeed, he only followed Hume, — that the judgment that a straight line is the shortest distance between two points is synthetic, as the notion of quantity is not to be elicited from the notion of 'straight' or of 'line' by analysis. If we regard the proposition from the standpoint of a man born blind, straightness would then only mean for us the sensation attendant on direct or uniform motion, such as the motion of the ulna on the humerus, and the comparative distance between two points could be understood, I imagine, as having a reference to nothing else but the temporal duration of the motion experienced in passing the finger along from one point to another in a surface, supposing such motion to be at a uniform rate. Repeated trials would soon form the basis of a generalization to the effect that, other things being equal, the more free from sensations of turning or wavering the motion was, the sooner would the goal be reached. Such a generalization would obviously be nothing else but a truth of experience. It is worthy of remark that straightness, like smoothness or purity, is based on the presence of one sensation, in this case a muscular one, and the absence of all others. It is this possession of a negative element that makes it capable of perfection. Roughness or crookedness, which are merely positive, have no limit to their degrees. The law, therefore, which connected the direct motion and brief duration of the journey between two points would not merely connect directness and brevity as such, but would embrace the proportional inference that the more nearly perfect the directness of the motion was, the briefer would the journey be from one point to another.

It will be observed that the law even in this elementary form has this important characteristic about it, that, so far as it goes, it is in the direction of introducing a quantitative standard for the conceptions based on muscular sensation, and it is a familiar truth that it is the introduction of a quantitative standard for sensations that brings the conceptions based on such sensations under the purview of exact science. The notion of quantity that we have so far reached, is itself, however, only of a rudimentary character, and lacks, indeed, the essential elements of the 'quantity' of science, namely, precision and objectivity. These come by measurement, and with measurement comes the developed conception of equality. Professor Bain, it seems to me, is clearly wrong when he says that the sole reason why we know that two things which are equal to the same thing are equal to one another, is that we have uniformly noticed this to be the case. It is rather because, for the purposes of geometry, we call no two things equal unless we assume them to have been measured with one another or with a third thing. Equality is not merely a case of perfect similarity in one respect; it is perfect similarity with a history; it is that sort of perfect similarity which is the result of a known cause. The notion of causality thus enters into it. On what, then, is the possibility of measurement itself dependent? It is dependent on the existence and recognition of the Thing, of that which is beyond all question the same thing for you as it is for me; and this, in the last resort, means that which I can put my finger on, and which you cannot put yours on without displacing mine. The muscular sensations that give us this experience are our ultimate warrant for regarding anything as the same and as permanent. Once we have the conception of the Thing in this sense, we can suppose that thing to be a foot-rule, and we can take measurements *ad libitum* which will bridge the gulf between us, and convert the subjects of such measurements into objective fact.

Supposing, then, that we have the edges of two surfaces to measure, and — still keeping to the blind man's standpoint — first run our finger along each to satisfy ourselves as to the absence of the sensations that accompany either roughness or curvature, and then place the surfaces side by side close to each other. If now we can run our finger along each end and experience the same absence of the sensation of unevenness, we pronounce the lines forming the two edges equal. It thus seems that, at bottom, straightness and equality are the same thing. A closer analysis would perhaps also satisfy us that parallelism is involved in the conception as well; that the lines which cross the ends of the straight lines that we measure, must not only be straight themselves but also parallel with each other.

We gain thus a notion of the equality that springs from coincidence, the equality that is perfect similarity both of size and shape. We have, however, in the conception of two similar periods of temporal duration, or of the same period of temporal duration for two people, combined perhaps with that of uniform motion during such a period, another sort of equality altogether ; and the identification of these two is part of the inductive process that precedes the developed conception of spatial equality. Having the concept of an equality that does not involve coincidence, we have a basis for the comparison of lines that do not coincide, and for pronouncing them equal or unequal to each other. Supposing them to be rigid, some third line that was bendable would have to be thought of as being used to measure with each. We would then find that, whenever we had one line with regard to which we were satisfied that its direction was uniform, and another line placed beside it joining the same points, the more closely the direction of the latter approximated to perfect uniformity, the less would be the unevenness between the end of its measure and the end of the straight line.

A man born blind might thus no doubt readily learn that a straight line is always the shortest distance between two points, and, in the case of the blind man born among a race of seeing men, so much of the concept of straightness would be imbibed with language that the proposition would probably seem self-evident. Whether a race of blind men would ever have arrived at it as a self-evident truth, would be an interesting inquiry if it could anyhow be answered. The reason, at any rate, why it seems self-evident to us is, I think, that we learn its truth and the grounds of its truth in learning to see. Visual straightness, itself a muscular sensation, is for us the mark of that which is the track of direct motion ; and visual length is the mark of the temporal duration of uniform motion. Vision thus in an instant coördinates for us the quality of the motion and its quantitative bearing.

When we speak of the quantitative aspect of geometry, we must distinguish between two things, namely, simple quantity and numerical quantity. Euclid does not introduce the latter, as regards angles at any rate. His conclusions are not, for all that, the less quantitative. The early geometricians did for the directions of straight lines what Descartes and Clairaut did for curves. The directions of straight lines corresponding to uniform forward motion, uniform sidewise motion, and so on, had in themselves nothing quantitative about them. It was the conception of the angle, and the predication in regard to one angle that it was equal to, or greater than, or less than another, that reduced them to a quantitative standard. An angle itself is nothing in the world; only straight lines and their directions exist. The fourth proposition determined the import of the equality of angles with each other, — in other words, the import of the quantitative inclination of one line to another; and it is on it that all the others rest. The parallelism of two lines means primarily their uniformity of direction. Direction, however, it must be observed, has itself no meaning except relatively to some standard. Our bodies are the ordinary standard which we use in practice. When we wish to make the proposition general and say of two lines that their direction is uniform, we must introduce the notion of an intersecting line as the standard of this uniformity. The simplest definition of the parallelism of two lines is that their inclination to an intersecting line is the same, and, when it is understood that we mean by inclination quantitative inclination, this definition is found to be identical with the definition that the exterior angle must be equal to the interior and remote.

If the view above taken of the nature of geometrical demonstration is at all correct, it is clear that the current opinion, given expression to by Mill among others, that the notion of cause and effect does not come into play in geometry, is very far from the truth. Geometry, indeed, is the science to which, I think, above all others we may look to give us a deeper insight than we at present possess into the true nature of causation. The consideration of any further suggestions, however, in connection with that aspect of the matter must be left till some other time.

WILLIAM W. CARLILE.